The center thickness of spectacle lenses

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The importance of the center thickness of spectacle lenses seems to be unquestionable. The weight, the power, the magnification and aberrations are all influenced by this parameler. Despite of its importance, it is really suprising how few are the publications on the subject in the ophthalmic literature.

The objective of this paper is to collect some methods available for the determination of the center thickness of the convex spectacle lenses and discuss them. The concave lenses will not be considered because they have a relative independent and standardized center thickness.

Let's analize the lens "a" of the figure 1. The front and back surfaces intersect each other at the periphery. The intersection of them defines the theoretical limit of the diameter of the lens. At this limit the lens will have an infinitesimal edge thickness or a "knife edge".

A knife edge lens is more important as a concept than as a physical entity. It is not only associated to the minimum diameter but also to the minimum center thickness. No practical spectacle lens can have a center thickness interior or even equal to it. If a lens is to have a center thickness smaller than the corresponding knife edge lens the diameter must be reduced. The inobservance of this fact often generates lenses impossible to be mounted on frame because of insuficient edge thickness or diameter.

For practical purposes the possibility of improper lens cut can be avoided by choosing a center thickness that is more than the antecipated minimum. This solution is far from the ideal because it adds superfluous thickness to these already thick lenses. Besides, it generates a great variability of thicknesses for lenses of the same configuration. A better approach is to try to find the appropriate thickness by mathematical analysis.

The center thickness varies as a function of every parameter of the lens: power, form, shape, index of refraction and edge thickness.

The form of the lens depends on the surface curvatures. It is associated to the terms bi-convex, plano-convex and meniscus.

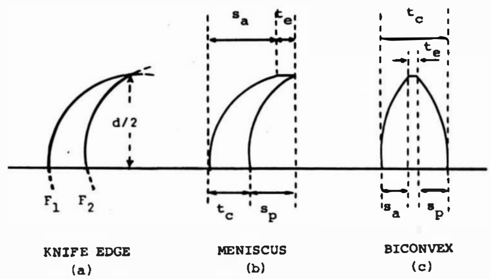

Fig. 1 — Sectional view of some one-half convex lenses. F_1, front surface power; F_2, back surface power; sa, front sag; sp, back sag; tc, center thickness; te, edge thickness.

The shape is determined by the design of the frame. It can be circular, oval, retangular, etc. It is a function of the pattern of distribution of the diameters of the lens.

Except for the power and edge thickness the influence of all other parameters may be studied as a unit employing the saggital depths or "sags" of the different curves of the lens. The saggital depth of a spherical surface is the distance from the apex to the base of the surface. The saggital value is obtained applying the theorem of Pythagoras in the triangle OCB of the figure 2. The resulting equation [2] is:

$$s^2 - 2rs + \left(\frac{d}{2}\right)^2 = 0 \qquad (1)$$

being $s \leq r$ the only root of our concern is:

$$s = r - \sqrt{r^2 - \left(\frac{d}{2}\right)^2} \qquad (2)$$

which may also be written as

$$s = \left(\frac{n'-n}{F_s} \cdot 10^3\right) - \sqrt{\left(\frac{n'-n}{F_s} \cdot 10^3\right)^2 - \left(\frac{d}{2}\right)^2} \qquad (3)$$

since

$$r = \left(\frac{n'-n}{F_s} \cdot 10^3\right)$$

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were s, d and F_s are respectively the saggital depth, cordal diameter and the power of the surface; n and n' are indices of refraction.

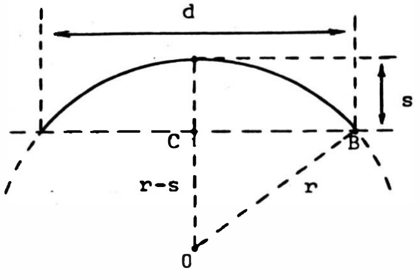

Fig. 2 — Sectional view of a spherical optical surface. s, saggital depth; d, cordal diameter; r, radius of curvature.

If the lens is a meniscus both sags are positive: for the convex surface n'−n and F_s are positive and for the concave surface both variables are negative. If the lens is bi-convex, the front sag is positive and the back sag is negative. The negative sign is due to a negative n'−n in the presence of a positve Fs.

Some of the saggital depths for curves in the range used for spectacle lenses are presented in the table I.

The relation of the center thickness to the saggital depths is better understood analysing the figure 1 (b and c). In these illustrations one should not consider the signs of the sags. If the lens is a meniscus the center thickness is given by the sum of the front sag and the edge thickness minus the back sag. If is bi-convex, the center thickness is given by the sum of both sags plus the edge thickness. This relationship may be expressed by the generic equation:

$$t_c = s_a \pm s_p + t_e \quad (4)$$

where t_c and t_e are the center and edge thicknesses and s_a and s_p the absolute values of the sags of the front and back surfaces. The plus sign is for plano and bi-convex lenses and the minus for the meniscus.

Unfortunately the equation 4 cannot be used directly. One of the sags is unknown until the corresponding surface is determined by calculation. The reasoning is that only one of the surfaces can be object of choice. The other must be calculated.

TABLE I

Some of the saggital depths for curves in the range used for spectacle lenses (for optical material with index of refraction equal to 1.523).

SURFACE POWER	CORDAL DIAMETER										
	32.0	36.0	40.0	44.0	48.0	52.0	56.0	60.0	64.0	68.0	72.0
1.0	0.2	0.3	0.4	0.5	0.5	0.6	0.7	0.9	1.0	1.1	1.2
2.0	0.5	0.6	0.8	0.9	1.1	1.3	1.5	1.7	2.0	2.2	2.5
3.0	0.7	0.9	1.1	1.4	1.7	1.9	2.3	2.6	3.0	3.3	3.8
4.0	1.0	1.2	1.5	1.9	2.2	2.6	3.0	3.5	4.0	4.5	5.0
5.0	1.2	1.6	1.9	2.3	2.8	3.3	3.8	4.4	5.0	5.7	6.4
6.0	1.6	1.9	2.3	2.8	3.4	4.0	4.6	5.3	6.1	6.9	7.8
7.0	1.7	2.2	2.7	3.3	4.0	4.7	5.4	6.3	7.2	8.1	9.2
8.0	2.0	2.5	3.1	3.8	4.6	5.4	6.3	7.3	8.4	9.5	10.8
9.0	2.2	2.9	3.5	4.3	5.2	6.1	7.2	8.3	9.6	11.0	12.5
10.0	2.5	3.2	4.0	4.8	5.8	6.9	8.1	9.5	10.9	12.6	14.4
11.0	2.8	3.5	4.4	5.4	6.5	7.7	9.1	10.7	12.4	14.3	16.5
12.0	3.0	3.9	4.9	6.0	7.2	8.6	10.2	12.0	14.0	16.3	19.0
13.0	3.3	4.3	5.3	6.5	7.9	9.5	11.3	13.4	15.8	18.7	22.3
14.0	3.6	4.6	5.8	7.2	8.7	10.5	12.6	15.1	18.1	21.9	27.4
15.0	3.9	5.9	6.3	7.8	9.6	11.6	14.1	17.1	21.0	27.1	-
16.0	4.2	5.4	6.8	8.5	10.5	12.9	15.8	19.7	26.0	-	-
17.0	4.5	5.8	7.4	9.3	11.5	14.3	18.2	23.9	-	-	-
18.0	4.8	6.2	8.0	10.1	12.7	16.1	21.3	-	-	-	-

For finding the power of the unknown surface there are the following equations derived from the back vertex power equation:

$$F_1 = \frac{(F_v - F_2)}{1 + \frac{t_c}{n} \cdot \frac{(F_v - F_2)}{10^3}} \quad (5)$$

$$F_2 = F_v - \frac{F_1}{1 - \frac{t_c}{n} \cdot \frac{F_1}{10^3}} \quad (6)$$

where F_1 and F_2 are the powers of the front and back surfaces and F_v the back vertex power.

The utilizatiion of one or another formula is a function of which surface is unknown. The corresponding sags are found using the equation 3. The problem is that these two formulas cannot be used directly either. They depend on t_c to be solved.

The conventional solution for this impasse is to make $t_c = 0$. The effect of the center thickness on the optic power is therefore completely neglected. The equations 5 and 6 are converted to:

$$F_{N1} = F_v - F_2 \quad (7)$$
$$F_{N2} = F_v - F_1 \quad (8)$$

where F_{N1} and F_{N2} are the values of F_1 and F_2 after nullifying t_c. Since they are different from their antecedents they are known as the front and back "nominal power" respectively. In other words, the unknown surface turns to be represented by its nominal power. The corresponding sag is calculated with Eq. 3 and t_c determined with Eq. 4.

Because of the desconsideration of the effect of the thickness on the optical power this method may be called the "Thin lens method". Thin lenses are those with irrelevant center thickness optical effect. In summary this method comprises the following steps:

1. determination of the sag of the known surface.
2. determination of the nominal power of the unknown surface.
3. determination of the sag of the unknown surface.
4. determination of the center thickness with Eq. 4.

The "real power" of the unknown surface can be found by substituting the value of t_c just determined in equation 5 and 6. Let us see one example: What should be the center thickness of a round convex lens with the following specifications: $F_v = +5.0$ D; $F_2 = -6.0$ D; d= 60.0 mm; $t_e = 1.5$ mm; n = 1.523?

1. the sag of F_2 — is found in table I, confronting 6.0 D with 60 mm. The result is, $s_p = 5.3$ mm.
2. the nominal value of the unknown surface, F_1 is:
   $F_{N1} = 5.0 - (-6.0) = 11.0$
3. the sag of F_{N1} — is found in table I, confronting 11.0 D with 60 mm. The result is, s = 10.7 mm.
4. the center thickness — since $F_2 < 0$, the lens is a meniscus the equation to be used is:
   $t_c = (10.7) - (5.3) + 1.5 = 6.9$ mm.

The real power of the front surface will be:

$$F_1 = \frac{(5.0 + 6.0)}{1 + \dfrac{6.9(5.0 + 6.0)}{10^3 \cdot 1.523}} = 10.48 \text{ D}$$

Being $F_{N1} > F_1$, s_a was overestimated. The resultant t_c is therefore larger than the necessary minimum.

A second and more precise solution for the impasse is the following: an initial guess for t_e is made and the value applied in the formulas 5 or 6 to find the power of the unknown surface. The sags are obtained with Eq 3 and the center thickness with the Eq. 4. If the calculated thickness is not significantly different from the guessed one the problem is solved; this is the minimum center thickness. If the difference is significant, the same sequence of procedures has to be repeated. The new guess may be the average of the last two values. The closer the initial guess is to the actual solution the faster the calculations will converge to an answer. Because of the repetitive and convergent nature of the method it may be called "Iterative method". All the nesty work may be done by a programmable calculator.

Using the data of the previous exemple, this method would provide the following results: $t_c = 6.25$ mm and $F_1 = 10.52$ D.

An analytic solution for the impasse could also be possible. One have three equations (3, 4 and 5) and three unknowns (t_c, F_s and s_a). It seems however that the probability of finding a simple algebric equation for t_c in this direction is rather small. Nevertheless this is an open field for further studies.

Instead of being solved the impasse may be avoided using the "Simplified method". It has three basic assumptions:

1. that the saggital depths of the surfaces of spectacle lenses are small (r is fairly large compared to d/2)
2. that the influence of the center thickness on the optical power is negligible (thin lens condition)
3. that the index of refraction is n = 1.500 The Eq. 1 may be rewritten as follows·

$$s = \frac{1}{2r} \cdot \left(\frac{d}{2}\right)^2 + \left(\frac{s^2}{2r}\right)$$

Because of the assumption 1, the second term of the above expression tend to be too small and may be neglected.
Hence,

$$s = \frac{1}{2r} \cdot \left(\frac{d}{2}\right)^2$$

which may be written as

$$s = \frac{F_s}{2\ (n' - n)\ 10^3} \cdot \left(\frac{d}{2}\right)^2$$

Substituting conveniently the above expression in Eq. 4 the center thickness is found to be:

$$t_c = \frac{F_1 \pm F_2}{2\ (n' - n)\ 10^3} \cdot \left(\frac{d}{2}\right)^2 + t_e$$

Accepting the assumption 2 the total power of the lens turns to be the algebric sum of the front and back surface powers:

$$F_v = F_1 \pm F_2$$

$$tc = \frac{F_v}{2\ (n' - n)\ 10^3} \cdot \left(\frac{d}{2}\right)^2 + t_e$$

After the assumption 3,

$$2\ (n' - n) = 2\ (1.5 - 1.0) = 1.0$$

hence,

$$t_c = \frac{F_v\ (d/2)^2}{10^3} + t_e \qquad (9)$$

The Eq. 9 is the final formula of the Simplified method. Using the data of the previous exemple the center thickness would be·

$$t_c = \frac{5\ (60/2)^2}{1000} + 1.5 = 6.0 \text{ mm}$$

Only for comparison, the minimum center thickness calculated with the Iterative method was 6.25 mm.

DISCUSSION

The three methods discussed above have something in common: they are all derived from the equation $s^2 - 2rs - \left(\frac{d}{2}\right)^2$ obtained by mathematical analysis of the figure 2.

The Thin lens and the Iterative methods are options for the solution of an impasse of two unknowns emerged during the calculations of t_c. In the former the impasse is solved by making the unknown surface to be represented by its nominal power. The corresponding sag will be larger than the real for convex surfaces and smaller for the concaves. As a consequence the center thickness is overestimated. In the later the problem is solved by iteration. The results are far more precise and under favorable conditions they provide the minimum center thickness.

The Simplified method is an alternative to avoid the impasse of the two unknows. The first assumption neglecting some terms tend to underestimate t_c. The second, tend to overestimate it for the same reason as the Thin lens method does. The third, has a variable effect depending on the index of refraction. The combination of all these tendencies limit the accuracy of the results.

If one is interested in precision he should choose the Iterative method. If a fast and very rough antecipation of the center thickness is wanted the Simplified method is a reasonable option. There is no advantage to determine the t_c with the Simplified method and then convert the results to the Thin lens method as proposed in the literature[3].

One important problem in all these methods is the identification of the meridian from which "F_v" and "d" should be taken.

It is beyond the scope of this paper to analyse in details this subject. Some suggestions however are given to facilitate the calculations.

The cases where the appropriate meridian can be found easily are of lenses of circular shape and/or of spherical power, both without decentrations.

If the lens has a circular shape and a spherical power, any meridian should serve since they are all equal each other. If it has a sphero-cylindrical power the choice must fall over the strongest. For lenses of non-circular shape and spherical power the meridian of choice should be the longest. These rules are to prevent the edge thickness to be inferior to the desirable value all the way around the lens.

Lenses of sphero-cylindrical power and a non-circular shape often pose some difficulties. The reasoning is that each meridian tend to have a different combination of power and cordal diameter. The strongest and the longest have been used supported more by empirical considerations than by scientific studies. As far a I know, there isn't yet a rule to find easily the meridian associated with the minimum center thickness of these lenses.

Surface curvatures greater than 18 D and center thickness over 18 mm are not practical[1]. A reasonable value for the average edge thickness is 1.5 mm. The distances of the equations 1 and 2 are measured in meters. For all other equations they are measured in millimeters.

SUMMARY

Three methods for determining the center thickness of convex spectacle lenses were analyzed. They all derive from the equations $s^2 - 2rs + (d/2)^2 = 0$, where d, r and s are respectively the cordal diameter, the radius of curvature and the saggital depth of the surfaces of the lens.

The Thin lens and the Iterative methods were devised to solve an impasse of two unknowns that emerged during the calculations of tc. The Simplified method is an alternative to avoid this impasse.

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KEY WORDS: center thickness, spectacle lenses, convex lens.

Seis pacientes com Oftalmia Simpática — Experiência de 13 anos

Lottenberg, C. L.[1]; Belfort Jr., R.[2]; Abreu, M. T.[3]; Domingues Neto, S.[4]; Hirata, P. S.[4]; Petrilli, A. M. N.[5]; Silva, M. H.[5]; Müller, M. E. W.[5]; Farah, M. E.[5]; Plut, R. C. A.[5]; Kim, M.[6]; Rehder, J. R. L.[7]; Burnier, M. N.[8]

INTRODUÇÃO

Oftalmia simpática é uma uveíte difusa, granulomatosa e bilateral que segue-se a um trauma ocular. Embora com mecanismo fisiopatológico desconhecido caracteriza-se clinicamente por um início insidioso e um curso prolongado sendo o período transcorrido entre o evento desencadeante e a sua manifestação como entidade clínica, bastante variável registrando-se extremos de 5 dias a 50 anos[1].

Apesar de muitos trabalhos existentes na literatura revista recentemente por Kraus — Mackiw[2], muitos pontos desta patologia continuam contraditórios, acreditando-se haver grandes variações geográficas e raciais[2,3]. No Brasil, de acordo com o dicionário biobibliográfico dos oftalmologistas, há 15 trabalhos publicados sobre oftalmia simpática entre 1870 e 1967 e nos últimos 20 anos apenas o trabalho de Alessandri, Oréfice e Miranda em 1982[5] que publicaram um caso clínico com documentação anátomo-patológica sobre oftalmia simpática ou irritação simpática.

Recentes trabalhos mostram que a oftalmia simpática continua a ser patologia importante, merecendo ser lembrada também em pacientes submetidos a intervenções cirúrgicas[2,3]. O objetivo do presente trabalho é de apresentar a experiência de seis casos dos serviços de uveíte da Escola Paulista de Medicina e da Faculdade de Medicina de Jundiaí em relação à oftalmia simpática no período compreendido entre 1972 a 1986, representando 0,4% da totalidade de 1.780 pacientes.

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